Why is the mastery of key competencies in mathematics for Indonesian students low? : Re-analysis of PISA 2012

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ABSTRACT

This study aims to determine the level of mastery of the key competency attributes of Indonesian students in mathematics. This study was approached quantitatively by adopting approach retrofitting (posthoc analysis). The data sources for this study were Indonesian students aged 15 years who took part in PISA 2012, as many as 5,622 students. The data of this research are ex post facto data obtained by documentation technique, as for what will be documented in the form of response data from Indonesian students based on the results of PISA 2012 and PISA 2012 instruments (item release PISA2012). The data analysis technique used is descriptive statistics using the DINA package R application. The results of this study indicate that Indonesian students are low in mastering the key competency attributes of mathematics related to mathematical operation (MO) and data analysis (DA); high in the mastery of key mathematical competency attributes related to mathematical abstraction (MA), logical reasoning (LR), mathematical modeling (MM), and intuitive imagination (II).

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1. Introduction

During the last 15 years, some scientific publications have attempted to discuss 21st-century skills. One of the skills currently being discussed is students' mathematical skills. This is based on previous research findings, which states that 21st-century society is a consumer of quantitative and sometimes statistical arguments (Wu, Zhang, Wu, & Chang, 2021). In a report on Basic Mathematics for the 21st Century, the National Council of Teachers of Mathematics (NCTM) points out that mathematical competence is the essential factor in influencing citizens' employment and income (Boesen, Lithner, & Palm, 2018).

Many previous studies have examined the importance of developing mathematical competence. For example, mathematical competence is associated with human resource development, maintenance of cultural identity, the pursuit of social reform, and creation of environmental awareness (Niss, 2015; Niss & Jablonka, 2020); the basis for assessment and decision-making to meet the needs of the 21st Century that is constructive, collaborative, and reflective (OECD, 2019); and an essential tool in problem-solving in various aspects of life (Kilpatrick, 2020). Indirectly, mathematical competence plays a critical role in achieving national and personal goals.

Mathematical competence itself, in its meaning, is defined differently by experts. Kilpatrick (2020) believes that mathematical competence is a tool for solving non-mathematical problems. Niss and Jablonka (2020) believe that mathematical competence is a practical experience that has implications for the social and cultural realm. Pettersen & Braeken (2019) believes that the definition of mathematical competence does not consider social and cultural differences. Overall, Economic Cooperation and Development (2019) defines mathematical competence as an individual's ability to reason mathematically and to formulate, use, and interpret mathematics to solve problems in various real-world contexts (OECD, 2019).

The Program for International Student Assessment (PISA) is a wide-scale assessment program that aims to measure students' mathematical competence (OECD, 2019). Indonesia is one of the countries that are members of PISA. The 2018 PISA results show that the mathematical competence of Indonesian students reached 379 with an OECD average score of 487 (OECD, 2019). This shows that Indonesia is in the low quadrant performance with high equity. In addition, this number can also be used as an indicator of errors made by Indonesian students in solving PISA mathematics competency questions.

Wu et al. (2021) believe that the mistakes made by students in solving the items describe the status of knowledge they do not have. In mathematics, the quality of knowledge possessed by these students is abstract and has a hierarchical structure. It is said to be abstract because it is identical to the material that uses many symbols in numbers, letters, or a combination of numbers and letters. At the same time, the hierarchical structure is related to the positioning of the sequence of knowledge status that students must learn and possess (Kartianom & Ndayizeye, 2017). This hierarchical structure and the relationship between knowledge states are learning trajectories (Duschl, Maeng, & Sezen, 2011).

The learning trajectory is a learning roadmap that strictly follows the laws of student knowledge. Tatsuoka (2009) defines the learning trajectory as a hierarchical structure of knowledge states that describes a partial order relationship between knowledge states. Informing the learning trajectory, students' understanding of concepts is assumed to follow the order of ease to difficulty, i.e. students first understand the basic attributes in the attribute hierarchy, then understand higher-order attributes, which are more difficult (Pettersen & Braeken, 2019). Therefore, to be able to know the cognitive law or the status of students' knowledge in more depth, diagnostic activities are needed.

Cognitive Diagnostic Models (CDM) is an interdisciplinary approach to diagnostic assessment. This model investigates the relationship between psychological processes and strategies that underlie the performance of items on a given test and the responses given by test takers to those items through sophisticated statistical analysis (De La Torre & Minchen, 2014). This model applies modern statistical thinking and cognitive theory to the psychological evaluation model, integrates the test objectives into the cognitive process model, and then reflects the psychological and cognitive characteristics of the subject (Rupp, Templin, & Henson, 2010). This model has the advantage of diagnosing the status of knowledge at detailed points, which in turn can increase students' understanding of mastery of knowledge and design scientific and reasonable learning and improvement programs.

Many previous studies have carried out research related to mathematical competence but more focused on discussing the problem of connotation and its conceptual definition. Meanwhile, the demands of learning in the 21st Century require students to have mathematical competence. In addition, previous research linking mathematical competence with diagnostic activities has also not been widely carried out, if any, the focus is only on comparing the accuracy of the diagnosis model by utilizing PISA items, not yet specific for diagnosing key mathematical competencies (Pettersen & Braeken, 2019; Wu et al., 2021). Likewise, research related to the use of diagnostic information for the needs of forming a mathematics learning trajectory is still very rarely done. Research so far has focused more on studying the learning trajectory on a small scale on certain mathematical materials (Rezky & Wijaya, 2018; Wu et al., 2021).

Based on some information from the research results and the PISA 2018 report that was discussed previously, it was found that there were difficulties experienced by Indonesian students in solving PISA questions. The results of previous studies have also not been able to provide more detailed information about the key competencies of Indonesian students in mathematics and the positioning of the learning sequence. Therefore, research related to the diagnosis of key competencies and the learning trajectory of Indonesian students' mathematics in solving PISA questions is very important to do. It is hoped that by conducting this research, information can be obtained that can be used as a basis for decision-making by the Government of Indonesia in terms of preparing Indonesian students for PISA 2021 and learning in the 21st-century era.

2. Method

2.1. Types of Research

This study was approached quantitatively by adopting approach retrofitting (posthoc analysis) (Ravand & Robitzsch, 2015). The approach retrofitting (posthoc analysis) is an approach that extracts response data from instruments non-test diagnostic into richer information. In addition, approach retrofitting (posthoc analysis) was also used to construct an instrument non-test diagnostic into a diagnostic test. The instrument non-diagnostic test used in this study is the 2012 PISA instrument.

2.2. Research Variables

PISA is held every three years to measure the mathematical, scientific, and language literacy of 15-year-old students. PISA 2012 focuses on measuring students' mathematical literacy, PISA 2015 focuses on measuring students' scientific literacy, PISA 2018 focuses on measuring students' language literacy, and the upcoming PISA 2021 will re-measure students' mathematical literacy (OECD, 2019). For PISA 2015, 2018, and 2021 the instrument could not be obtained or not available, only PISA 2012. Therefore, the data sources for this research were Indonesian students aged 15 years who took part in PISA 2012, 5,622 students.

2.3. Data Collection Techniques

As for what will be documented in the form of response data from Indonesian students based on the results of PISA 2012 and PISA 2012 instruments (item release PISA2012). The data of this research are ex post facto data obtained by documentation technique. Furthermore, a 2012 PISA instrument document review was conducted to obtain items that measure key mathematical competencies and the underlying cognitive attributes. The 2012 PISA instrument document review results then formed a Q-Matrix, which was used to connect the items with the underlying cognitive attributes.

2.4. Data Analysis Techniques

The data of this study were analyzed descriptively by using the DINA package R application. The use of R apps is free or open source. The results of data analysis using the DINA package R application provide information related to the characteristics of the PISA 2012 items and the profile of mastery of the key competency attributes of Indonesian students based on the diagnosis results using the DINA model (parameters slipping and guessing). In addition, the R application is also able to show the most dominant attribute pattern for the preparation of mathematics learning trajectories.

3. Results and Discussion

3.1. Result

3.1.1 Mastery Profile of Indonesian Students' Key Competency Attributes of Mathematics

The estimation method used in the DINA model is EAP (Expected a Posterior Estimation). This estimation method uses the posterior distribution to summarize the data and perform inferences by calculating the posterior mean or median. This estimation method is simple, efficient, stable, and usually produces better parameter estimates than those made by other methods (Chen, de la Torre, & Zhang, 2013). The results of estimating the probability of mastery of six key mathematical competencies of Indonesian students are presented in Table 1.

Table 1. Probability of Mastery of Indonesian Students' Mathematics Key Competency Attributes

Mathematics Key Competency Attributes	Attribute Mastery (%)
Mathematical Abstraction (MA)	58,73
Logical Reasoning (LR)	59,75
Mathematical Modeling (MM)	57,62
Intuitive Imagination (II)	54,24
Mathematical Operation (MO)	20,88
Data Analysis (DA)	23,63

Table 1 provides information that more than 50% of Indonesian students have mastered the attributes of Mathematical Abstraction (MA), Logical Reasoning (LR), Mathematical Modeling (MM), and Intuitive Imagination (II). This shows that Indonesian students have solved problems related to everyday life (non-mathematical) correctly. On the other hand, less than 25% of Indonesian students have mastered the attributes of Mathematical Operation (MO) and Data Analysis (DA). In other words, more than 25% of Indonesian students are not able to correctly complete the items that require attribute Mathematical Operation (MO) and Data Analysis (DA). This is very worrying because mathematical operations (MO) are a fundamental attribute in learning mathematics that plays an essential role in developing students' mathematical abilities. Likewise, with the mastery of data analysis (DA) attributes, because in this era of information disclosure (4.0), everything, both in policy formulation and in decision making, is based on data. To find out the causes of student errors in solving the PISA 2012 key competency questions, it can be identified through the percentage of latent classes presented in Figure 1.

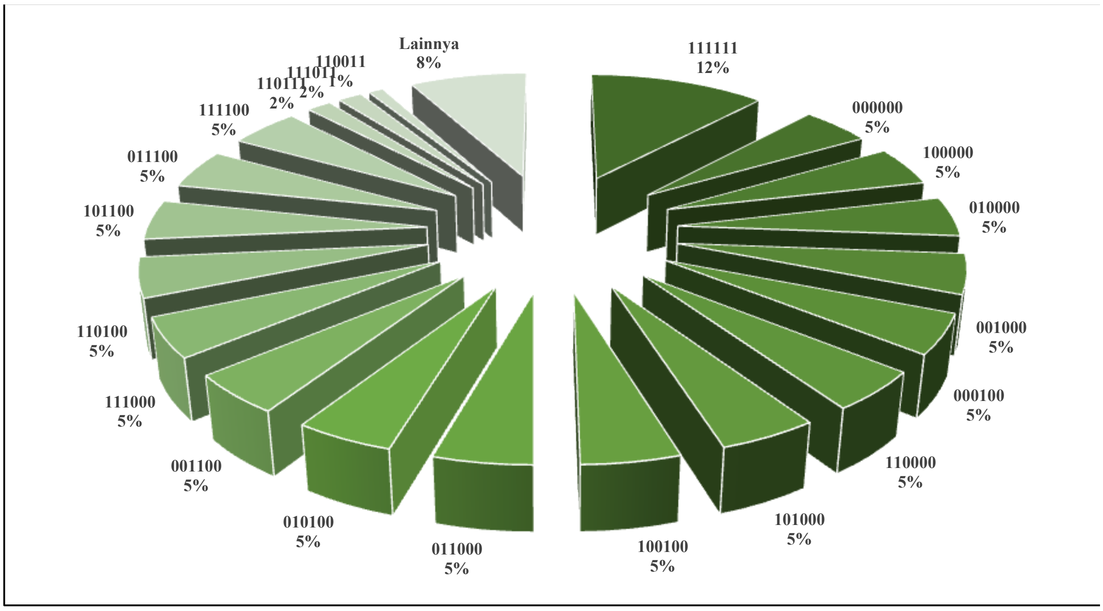

Fig. 1. Percentage of Latent Class based on Mathematics Key Competency Attributes

Based on the data presented in Figure 1, the percentage of the most dominant latent class is the latent class with a profile of 111111 of 12%. This means that 12% of Indonesian students have mastered all the attributes needed to complete 12 questions measuring the key competencies of PISA 2012 mathematics. Next is the latent class with profiles 000000, 100000, 010000, 001000, 000100, 110000, 101000, 100100, 011000, 010100, 001100, 111000, 110100, 101100, 011100, and 111100 have the same percentage, which is 5%. This can be interpreted that 5% of students do not master all attributes, 5% of students only master MA attributes, 5% students only master LR attributes, 5% students only master MM attributes, 5% students only master attribute II, 5% students master two attributes, namely MA and LR, 5% of students mastered two attributes, namely MA and MM, 5% of students mastered two attributes, namely MA and II, 5% of students mastered two attributes, namely LR and MM, 5% of students mastered two attributes, namely LR and II, 5% of students mastered two attributes, namely MM and II, 5% of students mastered three attributes, namely MA, LR, and MM, 5% of students mastered three attributes, namely MA, LR, and II, 5% of students mastered three attributes, namely MA, MM, and II, 5% of students mastered three attributes, namely LR, MM, and II, and 5% of students mastered four attributes, namely MA, LR, MM, and II. Let's look in more detail at the latent class with the dominant percentage. It can be concluded that the MO and DA attributes are the most dominant attributes that are not mastered by Indonesian students in solving 12 PISA 2012 key competency questions.

3.1.2 Analysis of Learning Trajectory for Indonesian Students' Key Competencies of Mathematics

3.1.2.1 Decision Tree C4.5

The learning trajectory of Indonesian students' mathematics in solving 12 items measuring the key competencies of PISA 2012 mathematics is divided into 12 tracks. These trajectories look very complex at the low level or level, while at the middle and high levels they are relatively simple. This can be interpreted that Indonesian students have different ways of solving the PISA 2012 key competency questions. For a more complete trajectory of Indonesian students' mathematics learning in completing the 2012 PISA key math competency items, it is presented in Figure 2.

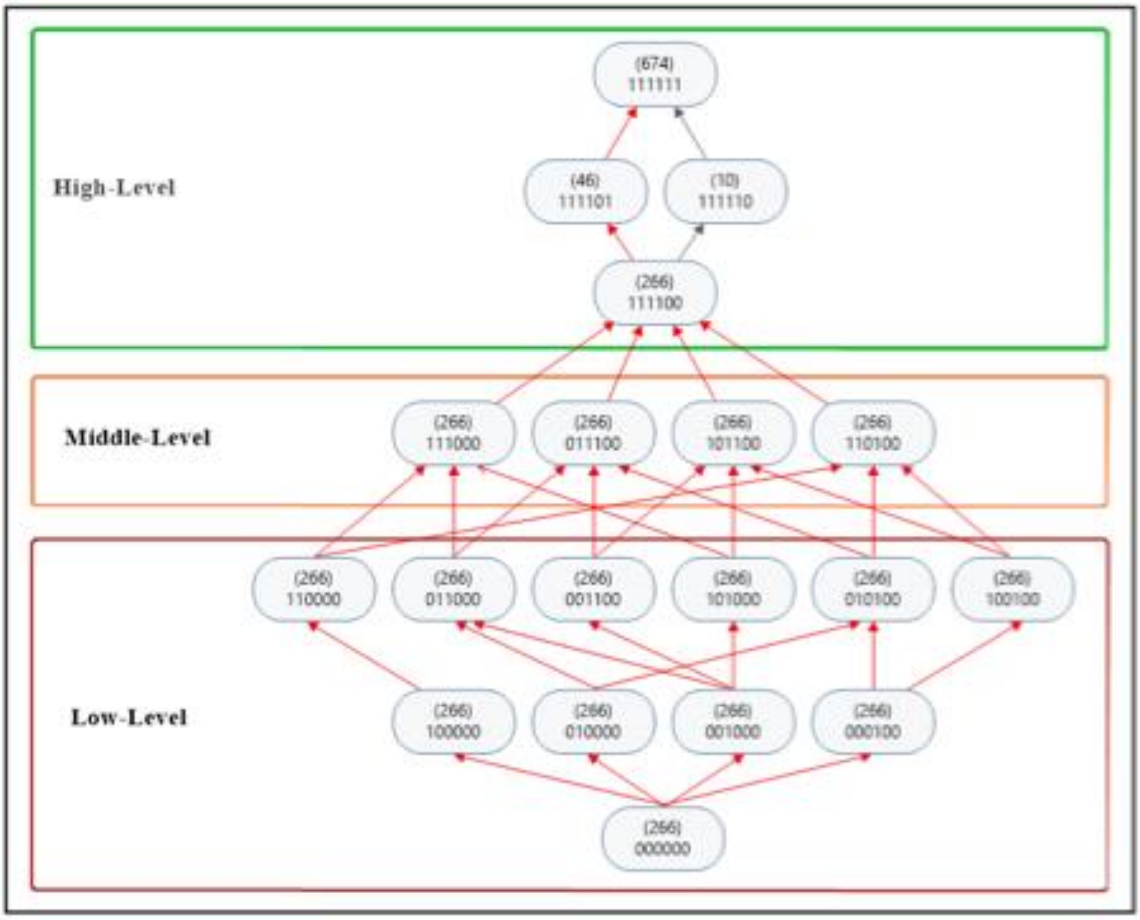

Fig. 2. Visualization of Decision Tree C4.5 Prediction Model

Based on the data presented in Figure 2, Indonesian students have different basic mathematical competencies. First, some students take MA as the basis of mathematical competence, then successively obtain LR, MM, II, DA, and finally obtain MO. Second, some students take LR as the basis of mathematical competence, then successively obtain MM, MA, II, DA, and finally obtain MO. Third, some students take MM as the basis of mathematical competence, then successively obtain LR, MA, II, DA, and finally obtain MO. Fourth, some students take II as the basis of mathematical competence, then successively obtain LR, MM, MA, DA, and finally obtain MO. Figure 2 also shows that the learning trajectory of Indonesian students has complex branches and has all levels or levels. This means that Indonesian students, apart from the 266 students who do not master all attributes, each student has at least one attribute of key mathematics competence. In addition, Figure 2 also shows that most of the education curriculum in Indonesia is still focused on low-level achievement, and most have also shifted to focus on achieving higher-order thinking skills.

3.2. Discussion

Data from the PISA test provides a platform for comparison and educational reference between different countries and regions. Usually, traditional research uses a variety of educational environments and other factors that influence education. The contents of the studies varied, some concerning student learning motivation, self-confidence, learning strategies, and environmental background information related to knowledge and skills, such as social, economic, cultural, educational background, demographics, and others (OECD, 2004). Currently, there are few dies on PISA content that relates information about PISA test questions and student score data. In this study, a cognitive diagnostic model was used to analyze the PISA test items and the results of students' answers, so as to integrate the assessment of core mathematics competencies into the cognitive diagnostic assessment process and extract key mathematical competencies (Wu, Wu, Chang, Kong, & Zhang, 2020; Wu et al., 2021). By using the existing measurement data, we can fully take advantage of students' mastery of the attributes that underlie the preparation of the questions.

Of the 12 items identified to measure students' key mathematical competencies, all of them matched the cognitive diagnostic model used, namely the DINA model. The DINA model is a cognitive diagnostic model that requires that all the attributes needed in solving questions must be mastered by students in order to solve them correctly. In addition, the DINA model also produces two parameters that can inform students' knowledge status. For example, from the guess (parameter guess), information is obtained that an average of 0.4% of Indonesian students who do not have all the attributes needed to complete the key math competency questions have the opportunity to answer correctly. So is the parameter slip(slip). We received information that an average of 64.7% of Indonesian students who have mastered all the attributes necessary to resolve key competencies item mathematical chance to answer incorrectly.

The high percentage of Indonesian students who slip in solving questions is more due to inaccuracy in performing mathematical arithmetic operations. This is evidenced by the low mastery of Indonesian students on the attributes of Mathematical Operation (MO) and Data Analysis (DA) (less than 20% of Indonesian students). Meanwhile, for other attributes such as Mathematical Abstraction (MA), Logical Reasoning (LR), Mathematical Modeling (MM), and Intuitive Imagination (II), less than 60% of Indonesian students have mastered or possessed it. The mastery of the key competency attributes of Indonesian students, when compared to their fellow countries from Southeast Asia, is still very much different. For example, in Singapore, more than 65% of students have mastered the attributes of Mathematical Abstraction (MA), Logical Reasoning (LR), Mathematical Modeling (MM), Intuitive Imagination (II), Mathematical Operation (MO), and Data Analysis (DA). This indicates that tIndonesia'sstrengthening and promotion of basic education and education reform is still very low. Singapore should have focused on strengthening and promoting basic education and education reform for a long time. Therefore, the future focus of the Indonesian government will be on improving students' understanding of mathematical operations, data analysis, logical reasoning, and intuitive imagination through curriculum reform and teaching arrangements.

Learning trajectories provide insight into students' thinking and learning in mathematics. It demonstrates the student's learning process through a series of tasks and promotes the development of psychological processes and students' thinking levels (Duschl et al., 2011; Jacobs, Mhakure, Fray, Holtman, & Julie, 2014; Tomul, Önder, & Taslidere, 2021). The learning trajectory plays an important role in the process of selecting learning objectives and learning methods. To achieve different learning objectives, appropriate learning objectives are needed. Different learning trajectories will determine the selection of different learning methods. Students who are indifferent learning trajectories are affected by the resources, and the learning environment received is also different. This supports the results of this study, which found that 12 main learning trajectories were formed in solving key mathematics competency items. The many main trajectories that are formed are closely related to the complex Indonesian education curriculum, the unequal competence of educators between schools, and the gap in facilities and infrastructure between schools.

Based on the study results, which showed a low level of mastery on the attributes of Mathematical Operation (MO) and Data Analysis (DA), institutional responsibility was needed to increase the level of mastery of the key competency attributes f Indonesian students in mathematics. The government's participation in reforming the curriculum and strengthening teaching is urgently needed to prepare graduates ready to compete in the 21st-century era. The government must also begin to take advantage of the results of large-scale assessments such as PISA, TIMSS, and National Assessments as a basis for policy formulation and decision making. Like the information obtained from the formed mathematics learning trajectory, the government can establish policies related to strategies, methods, and learning media that need to be used in teaching and learning activities in the classroom.

4. Conclusion

It turns out that the difficulty in learning mathematics experienced by students in several Southeast Asian countries during the last ten years is caused by the low level of student mastery of key mathematical competencies that require arithmetic operations and data analysis skills. The findings of this study (different from the findings of previous studies) indicate that students' learning difficulties in mathematics stem from factors in the educational curriculum that structure learning and learning outcomes. The number of learning materials that students must master with varying competencies of educators (teachers) has created gaps in problem-solving. Students who come from schools with the competence of educators (teachers) who are not following their expertise cannot solve problems that require basic mathematical abilities. The incompatibility of the competence of educators with their fields of expertise in teaching will bring risks to student learning outcomes. Students who are educated by educators who are not following their field of expertise will always lose achievement with students from schools with teachers following their field of expertise. In the end, they will also find it difficult to find work.

The DINA Cognitive Diagnosis Model used in this study has explained how Indonesian students' learning difficulties in mathematics are related to the complexity of the curriculum and national education settings. Students' learning difficulties in mathematics will not be resolved if curriculum reform is not carried out, especially in mathematics. At the same time, students' learning difficulties in mathematics also create a space for high graduate acceptance for students who come from schools with good educational settings. X. Wu et al. (2021) stated that curriculum reform is needed to focus on achieving mathematical competence to solve various practical problems in real life.

This study is limited to the data and cognitive diagnostic model used. This study uses PISA 2012 data. The Indonesian education curriculum has been reformed since 2016, so the information generated from this 2012 PISA data will experience a slight difference from the current Indonesian education context. This study only focuses on using the DINA cognitive diagnosis model, not comparing it with other diagnostic models such as DINO, G-DINA, G-DINO, LLCDM, and others. Such a diagnostic model allows for solutions to students' learning difficulties in mathematics to improve the mastery of more solutive key mathematical competencies. In line with that, further research is needed using the latest PISA data (eg, PISA 2022) using other diagnostic models such as G-DINA. In this way, comprehensive troubleshooting is possible.

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